

Online Appendix for  
**“Ambiguity Aversion and Household Portfolio Choice Puzzles:  
Empirical Evidence”**

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## Online Appendix A: Detailed description of the procedure for eliciting ambiguity aversion

This Appendix describes our procedure for measuring ambiguity aversion in the ALP survey.<sup>1</sup> The module starts with an introduction screen explaining the basic setup of the questions: see Figure A1. The introduction screen also explains that, after completing the survey, one of the respondent's choices in the set of three ambiguity gain questions will be selected randomly by the computer and played for a real reward of \$15.

*Figure A1 here*

In the next screen, shown in Figure 1 of the paper, the respondent is offered a choice between Box K, containing 50 purple and 50 orange balls, and Box U, containing an unknown mix of 100 purple and orange balls. Three response options are available: Box K, Box U, and Indifferent. If the respondent clicks the "Next" button before answering the question, the respondent is shown a message that all responses are important and is asked to answer the question again.

If the respondent selects "Indifferent", the matching probability ( $q$ ) is exactly 50% and the procedure continues with the second ambiguity question, described further on. If the respondent chooses Box K, she is ambiguity-averse and we know that the matching probability is less than 50% ( $0\% \leq q < 50\%$ ). In the following round, the number of winning balls in Box K is reduced to 25: see Figure A2. If the respondent selected Box U in the first round instead, she is ambiguity-seeking ( $0 \geq q > 50\%$ ) and in the second round the number of winning balls in Box K is increased to 75.<sup>2</sup>

*Figure A2 here*

The bisection algorithm continues this way for an additional three rounds (four rounds in total). In every round of the bisection algorithm, the difference between the lower bound and the upper bound on the matching probability is reduced by half.<sup>3</sup> When the option "Indifferent" is chosen, the algorithm stops earlier, as then the upper and lower bounds are equal. After a maximum of four rounds, we take the average of the lower and upper bound, the midpoint, as the estimate of the matching probability ( $q$ ). Table A1 shows all 27 possible outcome paths of the bisection algorithm, with corresponding matching probabilities. For two paths representing extremely ambiguity-seeking attitudes ( $q > 75\%$ , paths UUK and UUU) we require less measurement accuracy and the algorithm stops after three rounds to save time.

*Table A1 here*

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<sup>1</sup> Before including our survey module in the ALP panel, we piloted our questions in a laboratory experiment at the Wharton Behavioral Lab. Results of the lab experiment are available on request.

<sup>2</sup> An alternative would be to directly elicit the known probability that makes the subject indifferent between a known and an unknown lottery (e.g., Kahn and Sarin, 1988). We use a series of discrete choices as prior studies show this produces more reliable measures of preferences (e.g., Bostic, Herrnstein, and Luce, 1990; Noussair, Robbin, and Ruffieux, 2004).

<sup>3</sup> In theory, respondents could strategically increase the probability of winning \$15 by choosing Box U in the first round, thereby increasing Box K's known probability of winning in the subsequent rounds. The indicator variable is not subject to concerns about strategic answering as it depends only on the response in the first round, before the respondent could know that there are multiple linked rounds. Further, there is a maximum of four rounds, which limits the respondents' ability to learn strategic behavior.

## **2. Check questions to test for consistency of subjects' answers**

To test for the consistency of the answers we included two check questions. Using the answers to the ambiguity question (two ball colors, 50% ambiguity-neutral) we calculated the matching probability for each subject ( $q$ ). To generate Check Question 1, we lowered the known probability of winning for Box K to each subjects' matching probability minus 10 percentage points ( $q - 0.1$ ). In that case, the subject should choose the ambiguous Box U. To generate Check Question 2, we increased the known probability of winning of Box K to the matching probability plus 10 percentage points ( $q + 0.1$ ). In that case, the subject should choose the unambiguous Box K. Note that the maximum known probability is set at 99 and the minimum is set at 1, to avoid certainty.

**Table A1: Responses and matching probabilities for the ambiguity questions**

This table shows the possible outcomes in the four rounds of the ambiguity question, with two ball colors and initial 50% chance of winning for Box K. Panel A shows the transitions of the bisection algorithm, starting at Q1a, offering a choice between Box K with known winning probability  $p=50\%$  and ambiguous Box U. If the respondent chooses Box K, then next question round is Q1b (with  $p=75\%$ ), while round Q1i (with  $p=25\%$ ) follows after response Box U. After a choice of Indifferent, the algorithm always stops. Panel B shows the list of 27 possible response paths in the four question rounds. The letter combination in the column ‘Response’ summarizes one potential choices path, with K and U denoting the boxes, and I for Indifferent. The column  $q$  shows the corresponding matching probability, which is exact for paths ending with I and the average of the lower and upper bound for all other paths. For example, “KUUK” means the respondent chose Box K, followed by U twice, and then K. For this path the bounds on the matching probability are 38% and 44%, with midpoint  $q = 41\%$ . The path “I” represents an Indifferent choice in the first round ( $q = 50\%$ ). For paths UUK, UUI and UUU, extreme ambiguity-seeking, we require less accuracy and the algorithm stops after three rounds to save time.

<b>Panel A: Probability of Winning for Box K and Transitions</b>					
Question Round	Purple balls in Box K ( $p$ )	Orange balls ( $100 - p$ )	<u>Next round after response</u>		
			Box K	Box U	Indifferent
Q1a	50	50	Q1b	Q1i	stop
Q1b	25	75	Q1c	Q1f	stop
Q1c	12	88	Q1d	Q1e	stop
Q1d	6	94	stop	stop	stop
Q1e	18	82	stop	stop	stop
Q1f	38	62	Q1g	Q1h	stop
Q1g	32	68	stop	stop	stop
Q1h	44	56	stop	stop	stop
Q1i	75	25	Q1j	Q1m	stop
Q1j	62	38	Q1k	Q1l	stop
Q1k	56	44	stop	stop	stop
Q1l	68	32	stop	stop	stop
Q1m	88	12	stop	stop	stop

<b>Panel B: Outcome Paths</b>					
Response	$q$	Response	$q$	Response	$q$
KKKK	3	KUKI	32	UKKU	59
KKKI	6	KUKU	35	UKI	62
KKKU	9	KUI	38	UKUK	65
KKI	12	KUUK	41	UKUI	68
KKUK	15	KUUI	44	UKUU	71.5
KKUI	18	KUUU	47	UI	75
KKUU	21.5	I	50	UUK	81.5
KI	25	UKKK	53	UUI	88
KUKK	28.5	UKKI	56	UUU	94

**Figure A1: Screen shot: Text introducing the ambiguity questions**

You can win additional money on top of your regular payment for answering the survey, by answering the next questions.

You will be asked to choose between two boxes, Box K and Box U. Each box contains 100 balls of different colors. After you choose a box, one ball is drawn out of that box. If the ball is the right color, you could win \$15. There are no right or wrong answers for these questions. If you feel both boxes are equally attractive, please choose Indifferent.

After completing the survey, one of the questions you answered will be selected randomly by the computer and played for real money. Your winnings will be based on the choices you made.

[Next>>](#)

The logo for the RAND American Life Panel is centered on a yellow rectangular background. It features the word "RAND" in a serif font at the top, followed by a silhouette of a person sitting and using a laptop. Below the silhouette, the words "American Life" are written in a larger, bold serif font, and "Panel" is written in a smaller serif font at the bottom.

### Figure A2: Screen shot: Second round of the ambiguity question after Choice K

This Figure shows a screen shot from our ALP module, representing the second round of the ambiguity question. Box K now has a 25% known probability of winning; Box U has an unknown mix of balls with two different colors. After answering this question, respondents are led to a new round of the question. Selecting the "Indifferent" button finishes the question. If the respondent selects "Box K", he gets a new question with a lower probability of winning in Box K (fewer purple balls), while if he selects "Box U", the next question has a higher winning probability of winning in Box K (more purple balls).

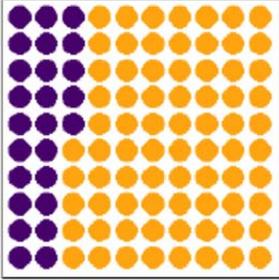
In the next question you can choose either Box K or Box U. Both hold 100 balls which can either be purple or orange.

For Box K, the exact mix of purple balls and orange balls is given below.  
Box U also holds purple and orange balls, but the mix is unknown.

In other words, both boxes hold 100 balls with two different colors (purple and orange). The mix of purple and orange balls is known for Box K and unknown for Box U.

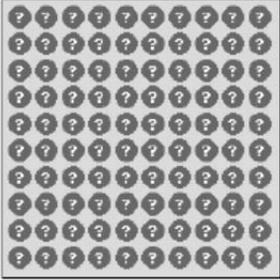
One ball will be drawn at random from the box you choose. You will win \$15 if a purple ball is drawn.

**Box K**



Chance	You win
<span style="color: purple;">■</span> 25%	\$15
<span style="color: orange;">■</span> 75%	\$0

**Box U**



Chance	You win
<span style="color: purple;">■</span> ?%	\$15
<span style="color: orange;">■</span> ?%	\$0

Box K       Indifferent       Box U

## Online Appendix B: The ALP survey and main control variables

### 1. Description of the American Life Panel

The American Life Panel (ALP) is an online panel of U.S. respondents age 18+; respondents were recruited in one of four ways (<https://mmicdata.rand.org/alp/>). Most were recruited from respondents to the Monthly Survey (MS) of the University of Michigan's Survey Research Center (SRC). The MS is the leading consumer sentiment survey that incorporates the long-standing Survey of Consumer Attitudes and produces, among others, the widely used Index of Consumer Expectations. Each month, the MS interviews approximately 500 households, of which 300 households are a random-digit-dial (RDD) sample and 200 are re-interviewed from the RDD sample surveyed six months previously. Until August 2008, SRC screened MS respondents by asking them if they would be willing to participate in a long-term research project (with approximate response categories "no, certainly not," "probably not," "maybe," "probably," "yes, definitely"). If the response category is not "no, certainly not," respondents were told that the University of Michigan is undertaking a joint project with RAND. They were asked if they would object to SRC sharing their information about them with RAND so that they could be contacted later and asked if they would be willing to actually participate in an Internet survey. Respondents who do not have Internet were told that RAND will provide them with free Internet. Many MS-respondents are interviewed twice. At the end of the second interview, an attempt was made to convert respondents who refused in the first round. This attempt includes the mention of the fact that participation in follow-up research carries a reward of \$20 for each half-hour interview.

Respondents from the Michigan monthly survey without Internet were provided with so-called WebTVs (<http://www.webtv.com/pc/>), which allows them to access the Internet using their television and a telephone line. The technology allows respondents who lacked Internet access to participate in the panel and furthermore use the WebTVs for browsing the Internet or email. The ALP has also recruited respondents through a snowball sample (respondents suggesting friends or acquaintances who might also want to participate), but we do not use any respondents recruited through the snowball sample in our paper. A new group of respondents (approximately 500) was recruited after participating in the National Survey Project at Stanford University. This sample was recruited in person, and at the end of their one-year participation, they were asked whether they were interested in joining the RAND American Life Panel. Most of these respondents were given a laptop and broadband Internet access.

### 2. Financial Literacy

The financial literacy questions we posed in the ALP module have been used in two dozen countries and comparable results obtained (Lusardi and Mitchell, 2011):

Suppose you had \$100 in a savings account and the interest rate was 2% per year. After 5 years, how much do you think you would have in the account if you left the money to grow?

- 1) More than \$102
- 2) Exactly \$102
- 3) Less than \$102
- 4) Don't know

Imagine that the interest rate on your savings account was 1% per year and inflation was 2% per year. After 1 year, would you be able to buy more than, exactly the same as, or less than today with the money in this account?

- 1) More than today
- 2) Exactly the same as today
- 3) Less than today
- 4) Don't know

Please tell us whether this statement is true or false. Buying a single company stock usually provides a safer return than a stock mutual fund.

- 1) True
- 2) False
- 3) Don't know

### **3. Trust**

The trust question we use was: “Generally speaking, would you say that most people can be trusted, or that you can’t be too careful in dealing with people? Please indicate on a score of 0 to 5.”). For the answers, we employ a Likert scale ranging from 0 to 5, with 0 indicating strong agreement and 5 indicating strong disagreement.

### **4. Risk Aversion**

To measure risk aversion, we build on Tanaka, Camerer, and Nguyen (2010), who ask respondents to select from a list consisting of 14 tradeoffs between two gambles. We modify their approach and use a sequence of binary choices similar to the method for eliciting ambiguity aversion described previously, as illustrated in Figure 2. If a respondent selects the certain outcome, he is then shown another choice with a higher expected value for the risky outcome. If he selects the risky outcome, he is then shown another choice with a lower expected value for the risky outcome. This process is repeated until risk aversion is sufficiently well-approximated. We use the responses to estimate each respondent’s risk aversion, measured as the coefficient of relative risk aversion assuming a power utility function.<sup>4</sup>

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<sup>4</sup> As in Tanaka, Camerer, and Nguyen (2010), the payoffs of the gambles are not integrated with total wealth in the utility function, and the power coefficient is limited to the range from 0 to 1.5. Risk aversion is defined as:  $1 - \text{power function coefficient}$ , and varies from -0.5 (risk seeking) to +1 (strongest level of risk aversion). A value of zero implies risk neutrality.

## Online Appendix C: Additional results

To further explore ambiguity attitudes across demographic and economic characteristics, Column (1) of Table C1 shows the results from regressing the ambiguity aversion measures on key control variables for the entire sample. Naturally the regressions do not imply any causal relation; rather, multiple regression is a convenient tool to concisely summarize the correlation structure of the data. In columns (2-4), we restrict the sample to certain groups of interest. Column (2) includes only respondents whose check question answers did not contradict their earlier choice. Column (3) includes only respondents with a college degree. Column (4) includes only respondents with at least \$500 in financial assets. The results are similar across columns.

*Table C1 here*

The results show that men are more ambiguity-averse than women. College-educated respondents are more ambiguity-averse than the less educated, suggesting that ambiguity aversion measures preferences rather than cognitive errors (i.e., such as cognitive errors due to using simplifying heuristics for complicated problems). There is also a positive relation between ambiguity aversion and risk aversion, consistent with Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010).

We also find that the survey question order matters: that is, measured ambiguity aversion proves to be *higher* when the risk aversion questions are presented *before* the ambiguity aversion questions. Such an order effect is consistent with the “comparative ignorance” hypothesis of Fox and Tversky (1995), which posits that ambiguity aversion is magnified by comparisons to less ambiguous events (in this case, the preceding risk questions with known probabilities). Because of this issue, we randomized the order of the risk and ambiguity questions in the ALP survey, and we also include an indicator variable for question order in the empirical analyses.

Perhaps the most striking aspect of Table C1 is that the adjusted R-square values are consistently low; the controls explain less than eight percent of the variance in ambiguity aversion. Even in column (2), in which the dependent variable likely contains less measurement error, the adjusted R-square is low. This suggests that our measure of ambiguity aversion captures new information about preferences which is not subsumed by standard demographic and economic controls.

**Table C1: Relation of ambiguity aversion with economic and demographic variables**

This table shows the results of OLS regressions in which the dependent variable is *Ambiguity Aversion* defined in Table 3 (50% -  $q$ ). The independent variables are defined in Table 1. Constant terms are included in the regressions, but not displayed in the interest of brevity. All non-binary independent variables are standardized to facilitate interpretation. Column (3) excludes respondents who gave inconsistent responses to either of the two check questions. Column (2) excludes respondents without a college degree. Column (4) excludes respondents with less than \$500 in financial wealth. Standard errors are clustered by household and appear in brackets.

	Full Sample (1)	Not Inconsistent (2)	College Educated (3)	Fin. Wealth $\geq$ \$500 (4)
<i>Age</i>	-0.325 * [0.19]	-0.280 [0.22]	-0.229 [0.22]	-0.620 ** [0.26]
<i>Age</i> <sup>2</sup>	0.265 [0.19]	0.243 [0.21]	0.145 [0.22]	0.581 ** [0.24]
<i>Male</i>	0.167 *** [0.05]	0.182 *** [0.05]	0.054 [0.06]	0.136 ** [0.05]
<i>White</i>	-0.152 ** [0.07]	-0.055 [0.07]	-0.137 [0.08]	-0.203 ** [0.09]
<i>Hispanic</i>	0.032 [0.07]	0.069 [0.08]	0.239 ** [0.12]	-0.021 [0.09]
<i>Married</i>	0.041 [0.06]	-0.003 [0.07]	0.032 [0.07]	0.058 [0.07]
<i>Number of Children</i>	0.009 [0.03]	0.011 [0.03]	-0.007 [0.04]	0.012 [0.03]
<i>Health</i>	0.007 [0.03]	-0.032 [0.04]	0.027 [0.03]	0.012 [0.04]
<i>High School</i>	0.097 [0.10]	-0.125 [0.12]		0.154 [0.20]
<i>College</i>	0.213 * [0.11]	0.028 [0.12]		0.264 [0.20]
<i>Employed</i>	-0.007 [0.06]	-0.017 [0.06]	0.125 * [0.07]	0.019 [0.06]
<i>Family Income</i>	0.048 [0.04]	0.059 [0.04]	0.011 [0.05]	0.047 [0.05]
<i>Wealth</i>	-0.010 [0.02]	-0.029 [0.03]	0.049 ** [0.02]	-0.018 [0.02]
<i>DC pension plan</i>	-0.009 [0.06]	-0.027 [0.06]	0.040 [0.07]	0.040 [0.07]
<i>DB pension plan</i>	-0.077 [0.07]	-0.103 [0.08]	-0.243 *** [0.09]	-0.093 [0.08]
<i>Financial Literacy</i>	-0.010 [0.03]	0.014 [0.03]	0.069 * [0.04]	-0.001 [0.04]
<i>Trust</i>	0.018 [0.02]	0.014 [0.02]	0.029 [0.02]	0.042 * [0.02]
<i>Risk Aversion</i>	0.179 *** [0.03]	0.121 *** [0.03]	0.146 *** [0.03]	0.133 *** [0.03]
<i>Question Order</i>	0.320 *** [0.05]	0.228 *** [0.05]	0.329 *** [0.06]	0.357 *** [0.06]
<i>Errors on Check</i>	-0.285 *** [0.05]		-0.284 *** [0.07]	-0.283 *** [0.06]
Adjusted-R <sup>2</sup>	0.093	0.052	0.105	0.100
N	2,972	1,766	1,182	1,884

Notes: \* significant at the 10%; \*\* 5%; and \*\*\* 1% level.

## Online Appendix D: Potential asset pricing implications

To demonstrate the effect of heterogeneity in ambiguity attitudes on the equity premium we use our estimates to calibrate the model by Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010). Table D1 shows the equity premium consequences in different types of economies. Column (1) shows the results for a representative agent expected utility (EU) economy as a benchmark: the single agent is neutral to ambiguity. In this case, the equity premium is 5.6%.<sup>5</sup> If, instead, the representative agent has a level of ambiguity aversion equal to the conditional mean in our group of ambiguity-averse respondents (*Ambiguity Aversion* = 0.17, Column 2), the resulting equity premium would be 10.1%. Thus, in principle, the ambiguity premium could be large. Alternatively, if the single agent had a level of ambiguity seeking equal to the conditional mean among our ambiguity-seeking respondents (*AA* = -0.19, Column 3), the equity premium would drop to only 2.4%.

*Table D1 here*

Given the heterogeneity in ambiguity attitudes in our ALP survey, the implied equity premium is 5.9% (Column 4). The ambiguity premium relative to the EU benchmark is only 0.5 percentage points, as the opposite demands of ambiguity-averse and seeking agents cancel out to a large extent. Finally in Column (5) we consider a market with a single agent with ambiguity aversion equal to the average ALP estimate (*AA* = 0.018). In such a case, the absolute price changes are of similar magnitude as in the heterogeneous agent economy, although slightly higher; the equity premium is 6.1%. Overall, these results provide suggestive evidence that the levels of ambiguity aversion we measure in the population are likely to have a limited effect on the equity premium.

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<sup>5</sup> The baseline equity premium is fairly high as, in the interest of simplicity, the model is not calibrated with consumption data and equity returns are the sole determinant of aggregate consumption.

**Table D1: Ambiguity aversion and the equity premium**

This table shows equilibrium asset prices in the model of Bossaerts et al. (2010). The  $\alpha$ -MaxMin model is used to represent ambiguity preferences, with  $\alpha \in [0, 1]$  the ambiguity aversion parameter. There are three possible states of the world: R, G and B. The probability of state R is known ( $\pi_R = 1/3$ ). The probabilities of states G and B are unknown ( $\pi_G, \pi_B \in [0, 2/3]$ , with  $\pi_G + \pi_B = 2/3$ ). Agents can trade Arrow-Debreu securities for the three states, with prices  $p_R, p_G$  and  $p_B$ . The aggregate endowment of the securities at time 0 is:  $e_R = 1, e_G = 1.2$  and  $e_B = 0.8$ . All agents perceive the same level of ambiguity and have the same level of relative risk aversion ( $\gamma = 2$ ), but they differ in their level of ambiguity. Our *Ambiguity Aversion* ( $AA$ ) measure is related to the ambiguity aversion parameter  $\alpha$  in the  $\alpha$ -MaxMin model as follows:  $\alpha = AA + 1/2$ . Column (1) shows results for a benchmark economy with a single ambiguity-neutral agent with  $\alpha = 1/2$  and prior probability  $\pi_G = 1/3$ , equivalent to an expected utility maximizer (EU). Column (2) shows results for an economy with one ambiguity-averse agent with  $\alpha = 0.67$  ( $AA = 0.17$ ), equal to the conditional mean among all ambiguity-averse ALP subjects. In Column (3) the economy has one ambiguity-seeking agent with  $\alpha = 0.31$  ( $AA = 0.19$ ), equal to the conditional mean among all ambiguity-seeking ALP subjects.. Column (4) shows the results for a heterogeneous 3-agent economy calibrated with our ALP estimates: the EU agent ( $\alpha = 1/2$ ) owns 10% of the aggregate endowment, the ambiguity-averse agent ( $\alpha = 0.67$ ) owns 52% and the ambiguity-seeking agent ( $\alpha = 0.31$ ) owns 38%. Column (5) shows prices for a single agent economy with ambiguity aversion equal to the ALP average ( $\alpha = 0.518$ , or  $AA = 0.018$ ). The columns labeled "% *chg*" show the percentage change in asset prices relative to the EU benchmark economy in Column (1).

	(1)	(2)	(3)	(4)	(5)
	1 EU agent	1 AA agent	1 AS agent	ALP Mix (EU, AS, AA)	ALP Average (1 Rep. AA agent)
<u>Agent distribution</u>	Proportion	Proportion	Proportion	Proportion	Proportion
% EU agents ( $\alpha = 0.5$ )	100%	0%	0%	10%	0%
% AA agents ( $\alpha = 0.67$ )	0%	100%	0%	52%	0%
% AS agents ( $\alpha = 0.31$ )	0%	0%	100%	38%	0%
% Rep. AA agents ( $\alpha = 0.518$ )	0%	0%	0%	0%	100%
<u>Equity premium</u>	5.6%	10.1%	2.4%	5.9%	6.1%

## Online Appendix References

- Bossaerts, P., Ghirardato, P., Guarnaschelli, S., Zame, W., 2010. Ambiguity in asset markets: theory and experiment. *Review of Financial Studies* 23, 1325-1359.
- Bostic, R., Herrnstein, R. J., Luce, R. D., 1990. The effect on the preference-reversal phenomenon of using choice indifference. *Journal of Economic Behavior and Organization* 13, 193-212.
- Fox, C. R., Tversky, A., 1998. A belief based account of decision under uncertainty. *Management Science* 44, 879-895.
- Kahn, B. E., Sarin, R. K., 1988. Modeling ambiguity in decisions under uncertainty. *Journal of Consumer Research* 15, 265–272.
- Lusardi, A., Mitchell, O. S., 2011. Financial literacy around the world: an overview. *Journal of Pension Economics and Finance* 10, 497-508.
- Noussair, C., Robbin, S., Ruffieux, B., 2004. Revealing consumers' willingness-to-pay: a comparison of the BDM mechanism and the Vickrey auction. *Journal of Economic Psychology* 25, 725–741.
- Tanaka, T., Camerer, C. F., Nguyen, Q., 2010. Risk and time preferences: linking experimental and household survey data from Vietnam. *American Economic Review* 100, 557-571.